A bivariate model for evaluating fair premiums of equity-linked policies with maturity guarantee and surrender option

M. Costabile\(^1\)  \quad I. Massabò\(^1\)  \quad M. Gaudenzi\(^2\)  \quad A. Zanette\(^2\)

\(^1\)Università della Calabria
\(^2\)Università di Udine

X Workshop on Quantitative Finance, Milan 2009
Outline

1. **Equity linked policies**
   - Introduction
   - Equity linked policies
   - Previous literature

2. **The bivariate model**
   - The model
   - Discretization of the bivariate model

3. **Calculus of the premium**
   - Singular points

4. **Numerical results**
Life insurance policies are usually long term contracts, the assumption of interest rates fixed during the whole policy lifetime is too restrictive. We consider a bivariate model that jointly describes the evolution of the equity price and of interest rates.

We adapt the approach proposed by Nelson-Ramaswamy [1990] and Wey [1996] by constructing a bivariate recombining tree derived after a discretization of the CIR model for the dynamics of interest rates as well as of the usual geometric Brownian motion describing the evolution of the equity value.

The reference found dynamic is not recombining and become unfeasible for very low number of steps. We use the technique of singular points Gaudenzi-Lepellere-Zanette [2007] in order to estimate of the premium.
Equity linked policies

- We consider an equity-linked policy whose payoff depends on the performance of a portfolio made up of equities of the same kind.
- The reference found is generated by investing a fixed contribution, $D$, to acquire equities at the beginning of each year until the maturity: $T$ years.
- We consider contracts whose payoff is generated by a reference found that depends on the underlying asset at given contribution times $t_i = i, i = 0, \ldots, T - 1$.
- At time $t_0 = 0$, with the first contribution $D$, the insurer buys $\frac{D}{S(0)}$ equities ($S(t)$ is the equity value at time $t$). At each contribution time $t_i$, the insurer buys $\frac{D}{S(t_i)}$ equities.
The reference found at time $t$, is therefore

$$F(0) = D, \quad F(t) = S(t) \sum_{j=0}^{j_0} \frac{D}{S(t_j)}, \quad t > 0$$

where $j_0$ is the largest index $j$ such that $t_j < t$.

Among the different types of minimum guarantees, we consider

$$G(t) = \sum_{j=0}^{j_0} De^{(t-t_j)\delta}, \quad G(T) = De^{\delta} \frac{e^{T\delta} - 1}{e^{\delta} - 1},$$

where $\delta > 0$ is the minimum guaranteed interest rate.

The payoff function is therefore

$$\phi(F(T), G(T)) = \max\{F(T), G(T)\}.$$
We consider a fixed term policy with **periodical premiums** $P$ to be paid at the beginning of each year but not at maturity.

A **surrender option** is embedded into the policy: the policyholder to escape out of the contract at the beginning of each year, just before the premium payment.

We consider the decision to surrender the policy as **endogenous to the evaluation model**, i.e., the policyholder decides to abandon the contract if this is financially convenient.

At maturity $T$, if the policyholder has not previously surrendered the contract, the insurer is forced to pay the maximum between the reference found value $F(T)$ and the minimum guarantee $G(T)$.
Equity linked policies

At every premium payment date \( t_i > 0, i = 1, \ldots, T - 1 \), the policyholder has two alternatives:

- to continue the contract paying the premium \( P \)
- to surrender the contract. Now the insured receives the maximum between the accrued reference found and the deemed contributions paid until \( t \) invested at rate \( \delta \):

\[
\phi(F(t_i), G(t_i)) = \max\{F(t_i), G(t_i)\}
\]

**Fair premium**

Given a fixed deemed contribution \( D \), for every choice of the premium \( P \) one has a fair present value \( V_0(P) \) (at time 0) of the policy contract previously defined. The *fair premium* of the contract is the value of \( P \) for which \( V_0(P) = 0 \).
Endowment equity-linked policy with a minimum guarantee and an embedded surrender option

- The policyholder pays a constant premium at the beginning of each year if he is alive at that date and has not been previously surrendered the contract.
- The insured has the right to early terminate the contract and this option may be exercised at the beginning of each year, just before the payment of the premium.
- If the insured dies during at time $t \in (0, T]$ the insurer pays at time $t$ the amount $\phi(F(t), G(t))$, closing the contract.
Previous literature

1. Seminal papers in case of minimum guaranteed
   - Boyle-Schwartz, The J. of Risk and Insurance, 1977

2. Some of the principal contributions
   - Aase-Persson, Scandinavian Actuarial Journal, 1994
   - Bacinello-Ortu, Insurance: Mathematics and Econ., 1993

3. Binomial techniques in a BS environment
   - Bacinello, Insurance: Mathematics and Economics, 2005
We consider the following dynamic for the equity value under the risk-neutral probability measure $Q$

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma_S dZ_S(t), \quad S(0) = S_0 > 0, \quad (1)$$

- $r(t)$ is the short rate,
- $\sigma_S$ is the constant stock price volatility,
- $Z_S(t)$ is a standard Brownian motion.

The risk-neutralized process for the short rate follows the CIR model:

$$dr(t) = k[\theta - r(t)]dt + \sigma_r \sqrt{r(t)} dZ_r(t), \quad r(0) = r_0 > 0. \quad (2)$$

- $k$ is the reversion speed,
- $\theta$ is the long term reversion target,
- $\sigma_r$ is the volatility of the interest process,
- $\rho$ is the correlation between $Z_r(t)$ and $Z_S(t)$.

If $\frac{2k\theta}{\sigma_r^2} \geq 1$ the trajectories of the process $r(t)$ are almost surely strictly positive.
A bivariate recombining lattice

- Following Wey [The J. of Financial Engineering, 1996], we can construct a bivariate recombining lattice describing the joint evolution of interest rates and equity value.
- An auxiliary process is defined which allows to deal with two orthogonal processes that can be approximated according to Nelson-Ramaswamy [The Review of Financial Studies, 1990] scheme.
- The orthogonality of the transformed processes allows us to build up a recombining bivariate lattice. This guarantees the tractability of the model from a computational point of view.
- We transform back the bivariate lattice into a new one to obtain the joint discretized evolution of the equity and of the spot interest rate.
First step: let us transform (1) and (2) into processes with constant (unit) volatility.

\[ X = \frac{\log S}{\sigma_S} \quad R = 2\sqrt{r}/\sigma_r. \]

By Ito’s Lemma

\[ dX(t) = \mu_X dt + dZ_S(t), \quad X(0) = \frac{\log S_0}{\sigma_S}, \]

and

\[ dR(t) = \mu_R dt + dZ_r(t), \quad R(0) = 2\sqrt{r_0}/\sigma_r. \] (3)

\[ \mu_X = \frac{r(t) - \sigma_S^2/2}{\sigma_S} \]

\[ \mu_R = \mu_R(R(t)) = \frac{k(4\theta - R(t)^2\sigma_r^2) - \sigma_r^2}{2R(t)\sigma_r^2}. \]
The model

Second step: Introduction of a process orthogonal to $R(t)$.

$$Y(t) = \frac{X(t) - \rho R(t)}{\sqrt{1 - \rho^2}}.$$ 

By standard calculations the processes $Y(t)$ and $R(t)$ have zero covariance and

$$dY(t) = \mu_Y dt + dZ_Y(t), \quad Y(0) = \frac{1}{\sqrt{1 - \rho^2}} \left[ \frac{\log S_0}{\sigma_S} - \frac{2\rho\sqrt{r_0}}{\sigma_r} \right],$$

(4)

where

$$\mu_Y = \mu_Y(R(t)) = \frac{\mu_X - \rho \mu_R}{\sqrt{1 - \rho^2}}$$

and $Z_Y(t)$ is a standard Brownian motion orthogonal to $Z_r(t)$. The underlying asset process $S$ and the interest rate process $r$ can be obtained from $Y, R$ using the relations:

$$r(t) \equiv \begin{cases} \frac{R(t)^2\sigma_r^2}{4} & \text{if } R(t) > 0 \\ 0 & \text{otherwise} \end{cases} \quad S(t) = e^{\sigma_S(\sqrt{1-\rho^2}Y(t)+\rho R(t))}.$$
Discretization of the bivariate model consistently with the Nelson-Ramaswamy binomial convergence approach

- The time to maturity $T$ is divided into $n$ intervals of length $\Delta t = T/n$.
- Since $R$ and $Y$ have unit variance, the size of each up step and down step are $\sqrt{\Delta t}$ and $-\sqrt{\Delta t}$, respectively.
- The size of the probabilities of up or down jumps has to be specified to assure the matching of the local drift and variance between the discrete and continuous model of $Y$, $R$. This guarantee that the discretized processes converge in distribution to the corresponding continuous time limit processes.
- To do this multiple jumps are needed to satisfy properly the matching conditions.
- We choose the approach of Li-Ritchken-Sankarasubramanian [J. of Finance ’95].
The discretized $R$-process, starting at time $t$ with value $R(t)$, at time $t + \Delta t$ may reach either $R^u$ (up jump) or $R^d$ (down jump). Local consistency conditions lead to

$$R^u = R(t) + M\sqrt{\Delta t}, \quad R^d = R(t) + (M - 2)\sqrt{\Delta t}.$$ 

$M$ is such that

$$R^d \leq R(t) + \mu_R \Delta t \leq R^u.$$ 

Hence $M$ is the smallest, odd integer $m : m\sqrt{\Delta t} \geq \mu_R \Delta t$.

The transition probability of an up jump, $q(R(t))$, is

$$q(R(t)) = \frac{\mu_R \Delta t + R(t) - R^d}{R^u - R^d},$$

the probability of a down jump is $1 - q(R(t))$.

We proceed in a similar way for the process $Y(t)$. Now, the transition probability of an up jump, $\hat{q}(R(t))$, becomes

$$\hat{q}(R(t)) = \frac{\mu_Y \Delta t + Y(t) - Y^d}{Y^u - Y^d}.$$
Figure: Multiple jumps for the discretized process $R(t)$. 
The bivariate tree

At each time step $i$, $i = 0, ..., n$, we consider $i^2$ nodes labelled $(i, j, k)$ corresponding to the time $i\Delta t$, to the values of $Y$ and $R$

\[
Y_{i,j} = Y_0 + (2j - i)\sqrt{\Delta t}, \quad j = 0, .., i
\]

\[
R_{i,k} = R_0 + (2k - i)\sqrt{\Delta t}, \quad k = 0, .., i.
\]

Starting from the node $(i, j, k)$, in consideration of possible multiple jumps and taking in account of the fixed tree structure, the process reaches the four nodes

\[
(i + 1, j_u, k_u), \quad \text{with probability } p_{uu},
\]

\[
(i + 1, j_u, k_d), \quad \text{with probability } p_{ud},
\]

\[
(i + 1, j_d, k_u), \quad \text{with probability } p_{du},
\]

\[
(i + 1, j_d, k_d), \quad \text{with probability } p_{dd},
\]
Discretization of the bivariate model

**Computation of** \( j_u, j_d, k_u, k_d \) **and** \( p_{uu}, p_{ud}, p_{du}, p_{dd} \)**

\( M \) is the smallest, odd integer \( m \) such that \( m \sqrt{\Delta t} \geq \mu_R(R_{i,k}) \Delta t \). \( M'(R_{i,k}) = \frac{M-1}{2} \)

\[
M_{i,j,k} = \begin{cases} 
  i - k & \text{if } M'(R_{i,k}) + k > i, \\
  -k & \text{if } M'(R_{i,k}) + k < 0, \\
  M'(R_{i,k}) & \text{otherwise} 
\end{cases}
\]

\[
p_{i,j,k} = \begin{cases} 
  1 & \text{if } M'(R_{i,k}) + k > i, \\
  0 & \text{if } M'(R_{i,k}) + k < 0, \\
  q(R_{i,k}) & \text{otherwise} 
\end{cases}
\]

In the same way, with respect to the \( Y \)-direction, we set \( \hat{M}(R_{i,k}) = \frac{M-1}{2} \) where \( M \) is the smallest, odd integer \( m \) such that \( m \sqrt{\Delta t} \geq \mu_Y(R_{i,k}) \Delta t \).

\[
\hat{M}_{i,j,k} = \begin{cases} 
  i - j & \text{if } \hat{M}(R_{i,k}) + j > i, \\
  -j & \text{if } \hat{M}(R_{i,k}) + j < 0, \\
  \hat{M}(R_{i,k}) & \text{otherwise} 
\end{cases}
\]

\[
\hat{p}_{i,j,k} = \begin{cases} 
  1 & \text{if } \hat{M}(R_{i,k}) + j > i, \\
  0 & \text{if } \hat{M}(R_{i,k}) + j < 0, \\
  \hat{q}(R_{i,k}) & \text{otherwise} 
\end{cases}
\]

The indexes of the multiple jumps as:

\[
j_d = j + M_{i,j,k}, \quad j_u = j_d + 1, \\
k_d = k + \hat{M}_{i,j,k}, \quad k_u = k_d + 1,
\]

and the joint probabilities:

\[
p_{uu} = p_{i,j,k} \hat{p}_{i,j,k}, \quad p_{ud} = p_{i,j,k} (1 - \hat{p}_{i,j,k}) \\
p_{du} = (1 - p_{i,j,k}) \hat{p}_{i,j,k}, \quad p_{dd} = (1 - p_{i,j,k})(1 - \hat{p}_{i,j,k}).
\]
Discretization of the bivariate model

Figure 1. The projection of the tridimensional bivariate tree on \((Y, R)\) plane
Singular points

Given a set of points: \((x_1, y_1), \ldots, (x_n, y_n)\), such that \(a = x_1 < x_2 < \ldots < x_n = b\) and

\[
\frac{y_i - y_{i-1}}{x_i - x_{i-1}} < \frac{y_{i+1} - y_i}{x_{i+1} - x_i}, \quad i = 2, \ldots, n - 1,
\]

let us consider the function \(f(x), x \in [a, b]\), obtained by interpolating linearly the given points.

We consider only piecewise linear functions with strictly increasing slopes, so that the function \(f\) is convex. The points \((x_1, y_1), \ldots, (x_n, y_n)\) (which characterize \(f\)), will be called the singular points of \(f\).
Let $f$ be a piecewise linear and convex function defined on $[a, b]$, and let $C = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ be the set of its singular points. Removing a point $(x_i, y_i)$ from the set $C$, the resulting piecewise linear function $\tilde{f}$, whose set of singular points is $C \setminus \{(x_i, y_i)\}$, is again convex in $[a, b]$ and we have:

$$f(x) \leq \tilde{f}(x), \quad \forall x \in [a, b].$$

**Figure:** $x_4$ has been removed.
LOWER BOUND

Let $f$ be a piecewise linear and convex function defined on $[a, b]$, and let $C = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ be the set of its singular points. Let $(\bar{x}, \bar{y})$ be the intersection between the straight line joining $(x_{i-1}, y_{i-1}), (x_i, y_i)$ and the one joining $(x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2})$. If we consider the new set of $n-1$ singular points

\[\{(x_1, y_1), \ldots, (x_{i-1}, y_{i-1}), (\bar{x}, \bar{y}), (x_{i+2}, y_{i+2}), \ldots, (x_n, y_n)\},\]

the associated piecewise linear function $\tilde{f}$ is again convex on $[a, b]$ and we have:

\[f(x) \geq \tilde{f}(x), \quad \forall x \in [a, b].\]

**Figure:** $x_3$ and $x_4$ have been removed, $\bar{x}$ has been inserted.
Singular points

**Calculus of the value of the contract**

Assume that the contribution $D$ and the premium $P$ are fixed.

At maturity at every node $(i, j, k)$ of the tree we can evaluate easily the minimum and maximum value

$$F_{i,j,k}^{\text{min}}, \quad F_{i,j,k}^{\text{max}}$$

of the reference found with respect to all possible paths of the underlying asset reaching the node $(i, j, k)$.

The policy price $v$, as function of the reference found $F$, is continuously defined by

$$v_{n,j,k}(F) = \phi(F, G(T)) = \max\{F, G(T)\}.$$ 

$v_{n,j,k}$ is a piecewise linear convex function characterized by the three singular points $(F^l_n, E^l_n)$, $l = 1, 2, 3$ ($F^l_n$ is the reference found, $E^l_n$ is the equity-linked policy price):

$$F^1_n = F_{n,j,k}^{\text{min}}, \quad E^1_n = G(T);$$
$$F^2_n = G(T), \quad E^2_n = G(T);$$
$$F^3_n = F_{n,j,k}^{\text{max}}, \quad E^3_n = F_{n,j,k}^{\text{max}}.$$
The value function at maturity
Consider now the step $i = n - 1$. A reference found $F$ at step $i$ generates 4 reference founds at step $i + 1$: $F_{uu}, F_{ud}, F_{du}, F_{dd}$.

$$F_{uu} = \begin{cases} \frac{FS_{i+1,j_u,k_u}}{S_{i,j,k}} & \text{if } i\Delta t \text{ is not a premium payment date,} \\ (F + D)\frac{FS_{i+1,j_u,k_u}}{S_{i,j,k}} & \text{if } i\Delta t \text{ is a premium payment date,} \end{cases}$$

Therefore

$$v_{i,j,k}(F) = e^{-r_{i,j,k}\Delta t}[p_{uu}v_{i+1,j_u,k_u}(F_{uu}) + p_{ud}v_{i+1,j_u,k_d}(F_{ud}) + p_{du}v_{i+1,j_d,k_u}(F_{du}) + p_{dd}v_{i+1,j_d,k_d}(F_{dd})].$$

Hence $v_{n-1,j,k}(F)$ is piecewise linear and convex as well. His singular values are $F_{min}^{n-1,j,k}$, $F_{max}^{n-1,j,k}$ (the extrema) and

$$G(T)\frac{S_{i,j,k}}{S_{i+1,j_u,k_u}}, G(T)\frac{S_{i,j,k}}{S_{i+1,j_u,k_d}}, G(T)\frac{S_{i,j,k}}{S_{i+1,j_d,k_u}}, G(T)\frac{S_{i,j,k}}{S_{i+1,j_d,k_d}}$$

if they belong to $(F_{min}^{n-1,j,k}, F_{max}^{n-1,j,k})$.

If $(n - 1)\Delta t$ is a premium payment date, these four values should be decreased of $D$.

In order to compute the corresponding policy prices we have to use the linearity.
We then proceed iteratively in the same way for $i = n - 2, \ldots, 0$. At the premium payment date $t_m$ an additional treatment is needed.

The payment of the premium $P$ reduces the policy price values of $P$. Therefore the new set of singular values becomes

$$(F_{i,j,k}^1, E_{i,j,k}^1 - P), \ldots, (F_L^1, E_L^1 - P).$$

In order to consider the surrender option, we have to check at time $t_m$ the convenience of the early exercise. The policy price function $v_{i,j,k}$ becomes

$$v_{i,j,k}(F) = \max\{\phi(F, G(t_m)), v_{i,j,k}^c(F) - P\},$$

where $v_{i,j,k}^c(F)$ is the continuation value computed previously. $v_{i,j,k}(F)$ is still piecewise linear and convex.

At the end we can compute $v_{0,0,0}(D) = E_{0,0,0}^1 - P$ which provides the pure lattice equity-linked policy value with $n$ steps, in the case of a periodic premium payment $P$, with a minimum guarantee and a surrender option.
Consider now an endowment equity-linked policy. If the insured dies during the interval of time \((i \Delta t, (i + 1) \Delta t)\) the insurer pays at time \((i + 1) \Delta t\) an amount

\[
\phi(F, G((i + 1) \Delta t)).
\]

Let \(t p_x\) the probability that an individual of age \(x\) survives the next \(t\) years, \(t q_x = 1 - t p_x\) represents the death probability.

If \(i \Delta t\) is not a premium payment date, the policy price function has to modified as follows:

\[
v_{i,j,k}^E(F) = \Delta t \ p_x i \Delta t v_{i,j,k}(F) + \Delta t \ q_x i \Delta t v_{i,j,k}^d(F)
\]

where

\[
v_{i,j,k}^d(F) = e^{-r_{i,j,k} \Delta t} [p_{uu} \phi(F_{uu}, G((i + 1) \Delta t)) + p_{ud} \phi(F_{ud}, G((i + 1) \Delta t)) + p_{du} \phi(F_{du}, G((i + 1) \Delta t)) + p_{dd} \phi(F_{dd}, G((i + 1) \Delta t))].
\]

At a premium payment date \(t_m\), \(v_{i,j,k}^E(F)\) becomes

\[
v_{i,j,k}^E(F) = \max \{ \phi(G(t_m), F), v_{i,j,k}^c(F) - P \}.
\]
The previous procedure provides the exact tree value with \( n \) steps of our equity linked policy.

However the number of singular points grows very rapidly and the procedure becomes unfeasible for small number of steps.

We are able to compute upper and lower bounds of the binomial price reducing drastically the amount of time computation and the memory requirement.

The distance of the upper and lower estimates from the pure binomial price can be controlled.
Remove $F_4$ if $\epsilon \leq h$. Inductively we get that the obtained upper estimate differs from the binomial value at most for $nh$. 
Remove $F_4$ and $F_5$ and insert $\bar{F}$ if $\delta \leq h$.

Inductively we get that the obtained lower estimate differs again from the binomial value at most for $nh$. 
This algorithm provides the estimate value at time 0 of the contract depending on the premium $P$, $\psi(P) = v_{0,0,0}(D)$.

Since the policy has to be fair, we must evaluate the premium $\bar{P}$ so that $\psi(\bar{P}) = 0$.

The equation is non linear and has to be solved numerically; its unique solution represents the fair periodical premium required by the insurer to write the contract.
Numerical solution of the equation $\psi(P) = 0$

- $\psi(P)$ is continuous, convex and strictly decreasing.
- For $P$ large enough $\psi(P)$ is linear.
Numerical solution of the equation $\psi(\bar{P}) = 0$

- $\psi(P)$ is continuous, convex and strictly decreasing.
- For $P$ large enough $\psi(P)$ is linear.
- The solution $\bar{P}$ of the equation $\psi(P) = 0$ is unique.
Numerical solution of the of the equation $\psi(\overline{P}) = 0$

- $\psi(P)$ is continuous, convex and strictly decreasing.
- For $P$ large enough $\psi(P)$ is linear.
- The solution $\overline{P}$ of the equation $\psi(P) = 0$ is unique.
- In order to solve this nonlinear equation we can use a secant algorithm with a suitable choice of the two initial points: $P_1$, where $\psi(P_1) > 0$, and $P_2$ where $\psi(P_2) < 0$. 
**Numerical solution of the equation** \( \psi(\bar{P}) = 0 \)

- \( \psi(P) \) is continuous, convex and strictly decreasing.
- For \( P \) large enough \( \psi(P) \) is linear.
- The solution \( \bar{P} \) of the equation \( \psi(P) = 0 \) is unique.
- In order to solve this nonlinear equation we can use a secant algorithm with a suitable choice of the two initial points: \( P_1 \), where \( \psi(P_1) > 0 \), and \( P_2 \) where \( \psi(P_2) < 0 \).
- We take \( P_1 \) as the premium of the corresponding equity-linked policy without surrender option. We can take

\[
P_2 = P_1 + \psi(P_1).
\]

\( \psi(P_2) \leq 0 \) because of the convexity of the function and the fact that for \( P \) large enough \( \psi(P) = \beta - P \), hence

\[
\psi(P) - \psi(P_1) \leq P_1 - P.
\]
### Numerical results

#### Case 1: Black-Scholes
- $S_0 = 100$, $\sigma_S = 0.1358$, $r_0 = 0.04$, $\theta = 0.04$, $k = 1$, $\sigma_r = 10^{-6}$, $\rho = 0$.

#### Case 2: positive correlation
- $S_0 = 100$, $\sigma_S = 0.1358$, $r_0 = 0.04$, $\theta = 0.04$, $k = 1$, $\sigma_r = 0.2$, $\rho = 0.25$,

#### Case 3: negative correlation
- $S_0 = 100$, $\sigma_S = 0.25$, $r_0 = 0.10$, $\theta = 0.1$, $k = 0.5$, $\sigma_r = 0.15$, $\rho = -0.2$.

We report the fair annual premiums for equity-linked endowment policies embedding a surrender option and a minimum guarantee for an individual initial age of 50 years. We model mortality by considering Italian Statistics for Male mortality in 2002.
## Policies embedding a surrender option.

### Estimate of the premium and maximal range of the error

<table>
<thead>
<tr>
<th>n</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T=5</td>
<td>T=10</td>
<td></td>
<td>T=5</td>
<td>T=10</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>106.806</td>
<td>106.800</td>
<td>106.782</td>
<td>108.202</td>
<td>108.193</td>
<td>108.193</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td>.000</td>
<td>.001</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>100</td>
<td>107.294</td>
<td>107.330</td>
<td>107.345</td>
<td>108.728</td>
<td>108.945</td>
<td>108.996</td>
</tr>
<tr>
<td></td>
<td>.002</td>
<td>.004</td>
<td>.032</td>
<td>.002</td>
<td>.005</td>
<td>.025</td>
</tr>
<tr>
<td></td>
<td>.002</td>
<td>.004</td>
<td>.029</td>
<td>.002</td>
<td>.005</td>
<td>.019</td>
</tr>
<tr>
<td>200</td>
<td>108.201</td>
<td>109.026</td>
<td>109.922</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.002</td>
<td>.031</td>
<td>.035</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMR</td>
<td>106.766</td>
<td>108.197</td>
<td></td>
<td>108.201</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BS</td>
<td>106.769</td>
<td>108.201</td>
<td></td>
<td></td>
<td></td>
<td>.001</td>
</tr>
</tbody>
</table>
Endowment policies embedding a surrender option with $x = 50$

Estimate of the premium and maximal range of the error

<table>
<thead>
<tr>
<th></th>
<th>T=5</th>
<th>T=10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>106.784</td>
<td>107.270</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td>.002</td>
</tr>
<tr>
<td>100</td>
<td>106.778</td>
<td>107.305</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td>.004</td>
</tr>
<tr>
<td>150</td>
<td>106.759</td>
<td>107.319</td>
</tr>
<tr>
<td></td>
<td>.001</td>
<td>.033</td>
</tr>
<tr>
<td>200</td>
<td>106.749</td>
<td>107.322</td>
</tr>
<tr>
<td></td>
<td>.001</td>
<td>.042</td>
</tr>
<tr>
<td>CMR</td>
<td>106,716</td>
<td></td>
</tr>
<tr>
<td>BS</td>
<td>106,746</td>
<td></td>
</tr>
</tbody>
</table>