The kernel methods and Riordan arrays: enumeration and asymptotics

Cyril Banderier

CNRS/Univ. Paris Nord

Many Riordan arrays, like many generating trees associated to some given rewriting rules can be also viewed as some lattice path models.

Here we will see how it is possible to make use of the kernel method, which is solving equations of the type

$$K(z,u)F(z,u) = \sum_i c_i(z,u)G_i(z)$$

where the G_i's and $F(z,u)$ are some unknown functions that we want to determine, and where the c_i's and $K(z,u)$ are some known coefficients. The methods consists in binding u and z such that $K(z,u) = 0$; this gives new equations (i.e., now, a system of equations for the G_i's) which allow to solve the initial equation. A nice feature of this method is that it is also giving access to many asymptotics properties, and limit laws.

We will illustrate this on Riordan arrays for quantities such that the asymptotics of the sums of values in each line, or the sum of all first lines. These are related to very interesting statistics on lattice paths, like the final altitude, or the area below these paths, and this is also related to the "internal path length" of the associated generating tree.

Few more questions that we tackle are: When the corresponding generating functions are rational/algebraic/transcendental? How is evolving the i-th cell of each line? When do we have a Rayleigh limit law? A Gaussian limit law? Are there some unimodular properties?

We will also mention the link with orthogonal polynomials (in link with height of lattice paths), and the case of exponential Riordan arrays.