Total Positivity of Riordan Arrays

Xi Chen

Dalian University of Technology
P. R. China

(Joint work with Huyile Liang and Yi Wang)

An infinite matrix is called totally positive if its minors of all orders are nonnegative. A nonnegative sequence $(a_n)_{n\geq 0}$ is called log-convex (log-concave, resp.) if $a_i a_{j+1} \geq a_{i+1} a_j$ ($a_i a_{j+1} \leq a_{i+1} a_j$, resp.) for $0 \leq i < j$.

The object of this talk is to study various positivity properties of Riordan arrays, including the total positivity of such a matrix, the log-convexity of the 0th column and the log-concavity of each row. We present sufficient conditions for the total positivity of Riordan arrays. As applications, two classes of special Riordan arrays are considered. One is related to the recursive matrices introduced by M. Aigner [1], the other is the consistent Riordan arrays introduced in [2]. We show that many well-known combinatorial triangles are totally positive and many famous combinatorial numbers are log-convex in a unified approach. In addition, we give a combinatorial interpretation of the log-convexity of the 0th column of a Riordan array.

References

[1] M. Aigner, *Catalan and other numbers: a recurrent theme*, in: Algebraic Combinatorics and Computer Science, Springer Italia, Milan, 2001, pp. 347–390.

[2] G.-S. Cheon, H. Kim, L. W. Shapiro, *Combinatorics of Riordan arrays with identical A and Z sequences*, Discrete Math. **312** (2012), 2040–2049.