Multidimensional Riordan arrays

Sung-Tae Jin

Department of Mathematics
Sungkyunkwan University
Suwon 440-746, Republic of Korea

(Joint work with Gi-Sang Cheon)

For the set of nonnegative integers $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$, let $\mathbf{N}_d = \prod_{j=1}^{d} \mathbb{N}_0$ for $d \geq 2$. A d-dimensional infinity matrix $M = [M(\mathbf{x}_d) | \mathbf{x}_d \in \mathbf{N}_d]$ over the complex field $\mathbb{C}$ is defined as a map

$$M : \mathbf{N}_d \to \mathbb{C}.$$

We can generalize rows and columns of a matrix as lines of a multidimensional matrix. A line of M is a set of positions $(x_1, x_2, \ldots, x_d)$ obtained by fixing $d-1$ of $x_1, x_2, \ldots, x_d$ and allowing the other index to vary in $\mathbb{N}_0$.

In this talk, we introduce the concept of multidimensional Riordan arrays. The lines of these arrays are defined by a finite sequence of generating functions over the ring of formal power series $\mathbb{C}[[z]]$. If the sequence is of two generating functions, the array is Riordan. We prove that the set of multidimensional Riordan arrays forms a group. Particularly, we focus on 3-dimensional Riordan arrays $D = [D(n, k, \ell)]$ defined as

$$D(n, k, \ell) = [z^n] g f^k h^\ell$$

where $g(0) \neq 0, f(0) = 0, h(0) \neq 0$ and $f'(0) \neq 0$. We then discuss a group extension problem of the Riordan group and their combinatorial interpretations.