On the multiple orthogonal polynomials of Sheffer type

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Let $\mathcal{P}$ be a vector space of polynomials over the complex field $\mathbb{C}$ and let $\mathcal{P}^*$ be its algebraic dual i.e., the set of all linear functionals $u : \mathcal{P} \to \mathbb{C}$. We denote $u(y)$ for $y \in \mathcal{P}$ by $\langle u, y \rangle$. Let $\{P_n\}_{n\geq 0}$ be a sequence of monic polynomials $P_n \in \mathcal{P}$ of degree n. For a positive integer d, the sequence $\{P_n\}_{n\geq 0} \subset \mathcal{P}$ is d-orthogonal if there exists a d-dimensional vector of linear functionals, $\mathcal{U} = (u_0, \ldots, u_{d-1})^T$, such that

$$\begin{cases} \langle u_k, P_r P_n \rangle = 0 & \text{if } r > nd + k, \quad n \in \mathbb{N} = \{0, 1, 2, \ldots\} \\ \langle u_k, P_n P_{nd+k} \rangle \neq 0 & \text{if } n \in \mathbb{N} \end{cases} \tag{1}$$

for each integer $k \in \{0, 1, \ldots, d-1\}$. A d-orthogonal polynomial sequence $\{P_n\}_{n\geq 0}$ is called of Sheffer type for (g, f) if there exist a pair of functions $g(z)$ and $f(z)$ where $g(0) \neq 0$, $f(0) = 0$ and $f'(0) \neq 0$ such that

$$g(t)e^{xf(t)} = \sum_{n\geq 0} S_n(x) \frac{z^n}{n!}.$$

In this talk, we consider d-orthogonal polynomial sequences $\{P_n(x)\}_{n\geq 0}$ of Sheffer type. Specifically, we discuss that

(i) a necessary and sufficient condition to be a Seffer type;

(ii) a d-dimensional vector of linear functionals for $\{P_n(x)\}_{n\geq 0}$;

(iii) the zeros of $\{P_n(x)\}_{n\geq 0}$;

(iv) the equivalence relation on the set of d-orthogonal polynomials of Seffer type.