# The Riordan Group in Several Variables

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(Joint work with Gi-Sang Cheon)

Let $z_1, \ldots, z_d$ be indeterminates and $\mathbb{C}[\![z_1, \ldots, z_d]\!]$ the ring of $d$-variate formal expressions of the form $\sum_{\mathbf{i}} a_{\mathbf{i}} \mathbf{z}^{\mathbf{i}}$ where $a_{\mathbf{i}} = a_{i_1,\ldots,i_d}$ and $\mathbf{z}^{\mathbf{i}}$ denotes the monomial $z_1^{i_1} \cdots z_d^{i_d}$ for $i_j \in \mathbb{N}_0$. Let $G$ be a $d$-variate generating function and $\mathbf{F} = (F_1, \ldots, F_d)$ be a vector function of $d$ generating functions in any number of variables, all with vanishing constant terms. Define the formal composition by $G \circ \mathbf{F} = G(F_1, \ldots, F_d)$.

In this talk, we consider the set $\{(G, \mathbf{F})\}$ where $G(\mathbf{0}) \neq 0$ and the Jacobian determinant of $\mathbf{F}$ is not zero i.e., $\det\left(\frac{\partial F_j}{\partial z_i}\right) \neq 0$. Then we show that the set $\{(G, \mathbf{F})\}$ forms a group under the binary operation

$$(G, \mathbf{F})\,(H, \mathbf{L}) = (G(H \circ \mathbf{F}), \mathbf{L} \circ \mathbf{F})\,.$$

Furthermore, we give a matrix representation for the group element $(G, \mathbf{F})$ which will be called a $d$-Riordan matrix. All its entries might be expressed in terms of the form

$$\left[z_1^{i_1} \cdots z_d^{i_d}\right] G\left(\mathbf{z}\right) F_1^{j_1}\left(\mathbf{z}\right) \cdots F_d^{j_d}\left(\mathbf{z}\right),$$

and the $d$-Riordan matrix is not necessarily lower triangular.