KRYLOV MATRICES VERSUS ORTHOGONAL POLYNOMIALS

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(Joint work with Gi-Sang Cheon)

Let $(p_n(x))_{n\geq 0}$ be a sequence of orthogonal polynomials p_n of degree n satisfying the three term recurrence relation:

$$p_{n+1}(x) = (x - a_n)p_n(x) - b_n p_{n-1}(x)\,.$$

The Hankel determinant associated to the sequence $(p_n(x))_{n\geq 0}$ is known as

$$\det H = b_1^n b_2^{n-1} \cdots b_n^1 \tag{1}$$

for any sequence (a_n).

In this talk, we are interested in an $n \times n$ matrix whose determinant can be expressed as the form (1). We show that if A is a lower triangular matrix whose diagonal entries are the same then the Krylov matrix defined as $K(A, \mathbf{v}) = \left[\begin{array}{ccccc} \mathbf{v} & A\mathbf{v} & A^2\mathbf{v} & \cdots & A^{n-1}\mathbf{v} \end{array}\right]$ has

$$\det(K(A, \mathbf{v})) = v_1^n a_{2,1}^{n-1} a_{3,2}^{n-2} \cdots a_{n,n-1}^1 \tag{2}$$

for any vector $\mathbf{v} \in \mathbb{R}^n$. For example, if A is the Pascal matrix, a Riordan matrix of the form $\left(\frac{1}{1-z}, \frac{z}{1-z}\right)$, and $\mathbf{v} = (1, 1, \ldots, 1)^T$, then $K(A, \mathbf{v})$ is the Vandermonde matrix with the determinant $1^n 1^{n-1} 2^{n-2} \cdots (n-1)^1$. This is motivated from the geometric progression matrices. In addition, we discuss how Krylov matrix $K(A, \mathbf{v})$ with the determinant (2) can be contributed to the orthogonal polynomials.